Progression in Calculation

Babcock 4S

Aims

The national curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Introduction

Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording. Skills need to be taught, practised and reviewed constantly. These skills lead on to more formal written methods of calculation.

Strategies for calculation need to be represented by models and images to support, develop and secure understanding. This, in turn, builds fluency. When teaching a new strategy it is important to start with numbers that the child can easily manipulate so that they can understand the methodology.

The transition between stages should not be hurried as not all children will be ready to move on to the next stage at the same time, therefore the progression in this document is outlined in stages. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

A sound understanding of the number system is essential for children to carry out calculations efficiently and accurately.

Magnitude of Calculations

Year 1 – U + U, U + TU (numbers up to 20) including adding zero, U – U, TU – U (numbers up to 20) including subtracting zero, U x U, U ÷ U

Year 2 - TU + U, TU + multiples of 10, TU + TU, U + U + U, TU - U, TU – tens, TU – TU, TU x U, U ÷ U

Year 3 – add numbers with up to three-digits, HTU + multiples of 10, HTU + multiples of 100, subtract numbers up to three-digits, HTU – U, HTU – multiples of 10, HTU – multiples of 100, HTU – HTU, TU x U, TU ÷ U

Year 4 - add and subtract numbers with up to four-digits, ThHTU + ThHTU, ThHTU - ThHTU, add and subtract decimals with up to two decimal places in the context of money, multiply three numbers together, TU x U, HTU x U, TU x U, multiply by zero and one, TU ÷ U, HTU ÷ U

Year 5 – add and subtract numbers with more than four-digits, add and subtract decimals with up to three decimal places, ThHTU x U, ThHTU x TU, HTU x TU, multiply whole numbers and decimals with up to three-decimal places by 10, 100 and 1000, divide numbers with up to four-digits by U (including remainders as fractions and decimals and rounding according to the context)

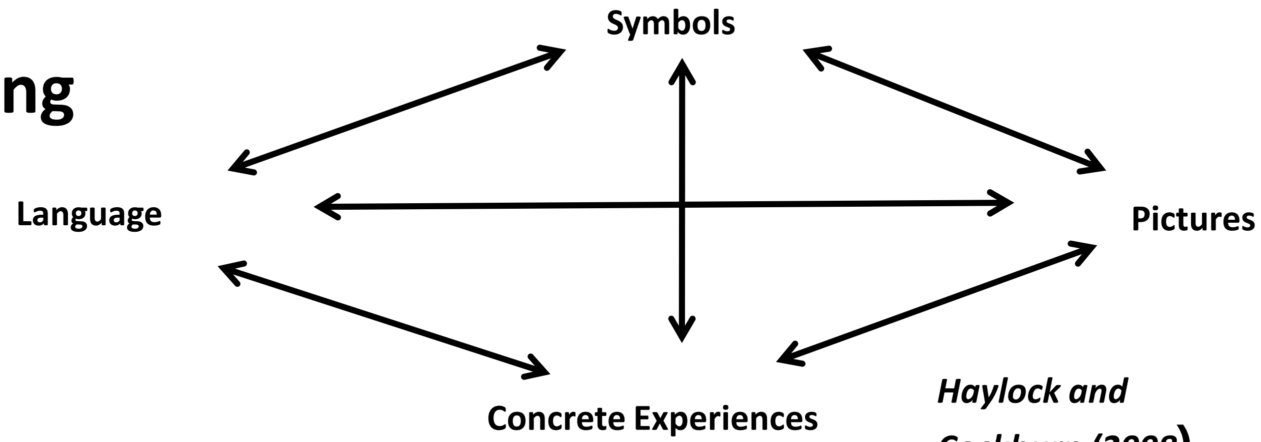
Year 6 - add and subtract numbers with more than four-digits, add and subtract decimals with up to three decimal places, multiply numbers with up to four-digits by TU, multiply numbers with up to two-decimal places by a whole number, divide numbers up to four-digits by TU (interpreting remainder according to the context), divide decimals up to two-decimal places by U or TU

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. … pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects.

National Curriculum 2014

Structuring Learning

Children ***must*** *have concrete experiences that enable them to create visual images. They should be encouraged to articulate their learning and to become pattern spotters.*



Symbols

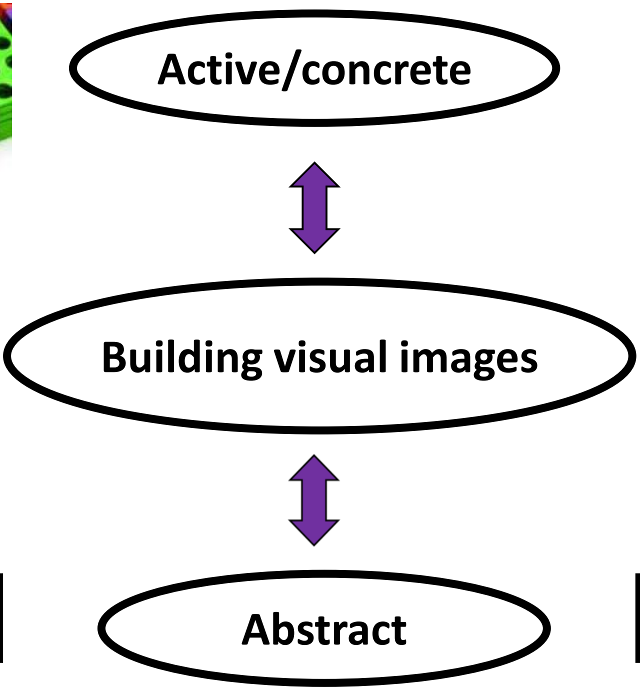
Language

Pictures

Concrete Experiences

Haylock and Cockburn (2008)

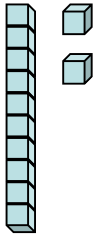
Communicating Mathematically

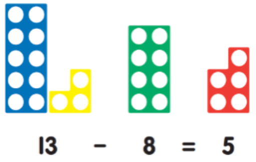


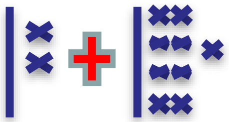
Active/concrete

Building visual images

Abstract







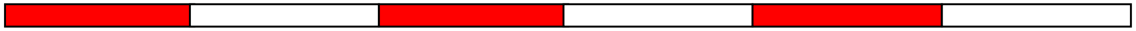
13 - 8

12 + 19

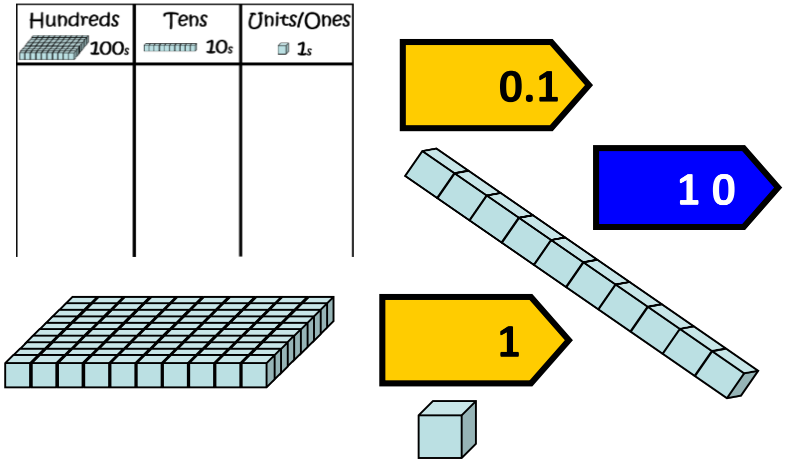
Pattern Spotting

bead string

count stick

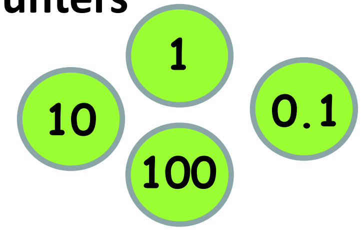


place value apparatus



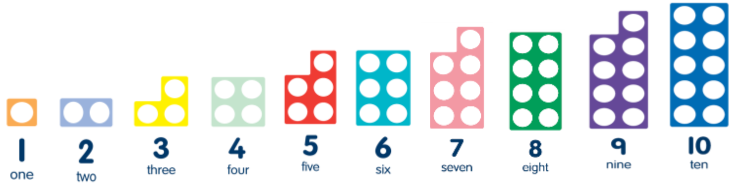
Multilink

place value counters

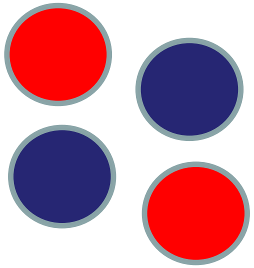


Cuisenaire

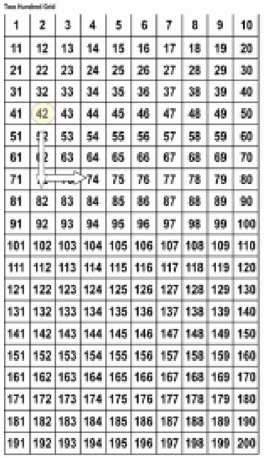
Numicon



double sided counters



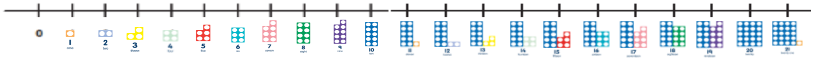
number grids 100 and 200



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	[illegible]	[illegible]	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	[illegible]	[illegible]	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

number line

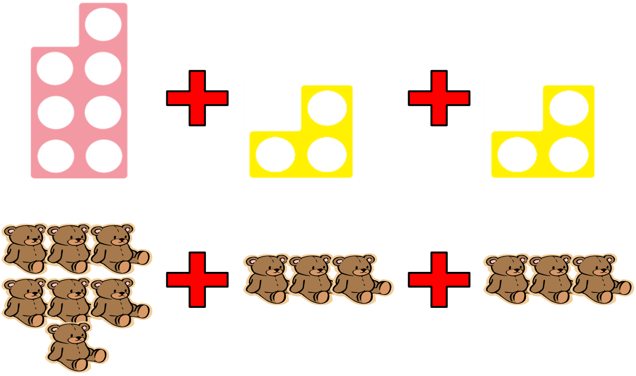


Structures of Addition (Haylock and Cockburn 2008)

Children should experience problems with all the different addition structures in a range of practical and relevant contexts e.g. money and measurement

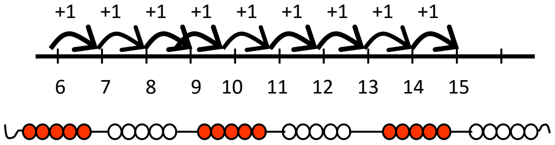
Aggregation

Union of two sets
How many/much altogether?
The total



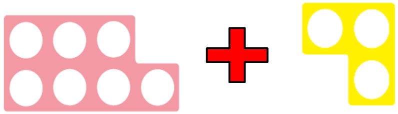
Augmentation

Start at and count on
Increase by
Go up by

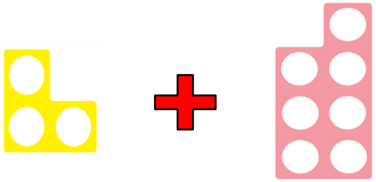


Commutative law

Understand addition can be done in any order
Start with bigger number when counting on
(Explain to children that subtraction does not have this property)



is the same as/equal to (=)



Addition

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.
Addition and subtraction should be taught together.

End of Year Expectations

Possible Concrete and Visual Representations

Children's Recording

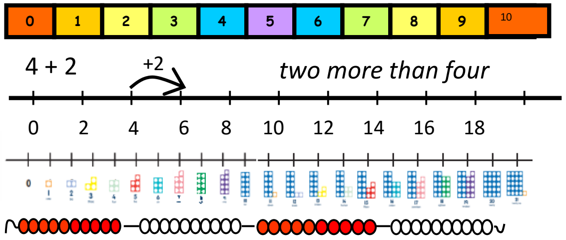
Fluency

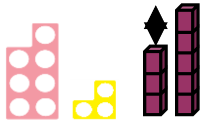
Year 1

Children must experience combining two, **and then more than two**, groups of objects using counting on and the language of addition e.g. add, plus

Children must experience increasing numbers e.g. what is two more than seven ?

Compare quantities to say how many less and/or how many more





'two more than three is five or two less than five is three'

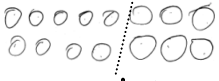
Use practical resources such as bears, counters, cubes and number lines/hundred grids and progress to a resource such as Numicon to encourage counting in groups rather than ones

If using Numicon, children could use printed Numicon icons and stick these in - progressing to recording number sentences alongside

1 + 2 = 3

Children may record pictorially progressing to recording number sentences alongside

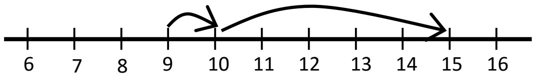
Example



9 and 6

9 + 6

9 + 6



Count forwards, to and across 100, beginning with 0 or 1 or from any given number

Switch count between tens and ones e.g. 10, 20, 30, 31, 32, 33 …

Represent and use number bonds up to 20 (establish addition and subtraction as related operations)

Find one more than a number

Find ten more than a number

Count in multiples of 2s, 5s and 10s starting on multiples to highlight pattern recognition

Year 2

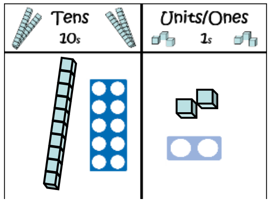
Children should be able to partition numbers in different ways e.g. as 2+2+2+1 or 5+3 or 23 as 20 +3 or 10+13

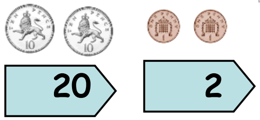
Children should use concrete objects, pictorial representations and add numbers in different contexts e.g. money, measures

ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE** THAN TWO NUMBERS

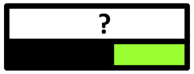
Children should understand the language of sum

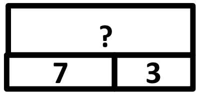
Ensure children understand that addition is commutative (can be done in any order)



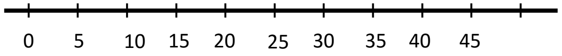


Cuisenaire

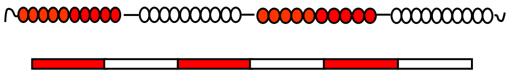




Bar Model



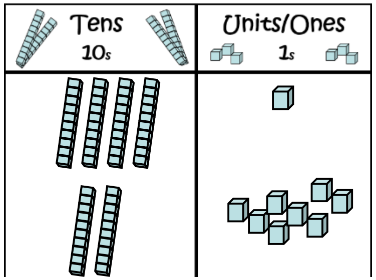
Numbered and ***partially numbered*** number lines



Use Numicon, number grids, place value apparatus/Dienes, place value grids, place value cards, Encourage children to partition numbers rather than counting in ones.

Children apply, develop and secure their understanding of place value

Use jottings and record number sentences



41

\+

28

40	1
+ 20	+ 8
= 60	= 9

60 + 9 = 69

Show increasing fluency in deriving pairs of numbers up to 10 and then up to 20

Use knowledge to derive and use number facts up to 100

Add numbers mentally including TU + U, TU + tens, TU + TU, U + U + U

End of Year Expectations

Year 3

Add numbers with up to three-digits

(leading to formal written column method)

ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN** TWO NUMBERS WITH DIFFERING NUMBERS OF DIGITS

Children should partition numbers, up to 1000, in different ways

e.g. 100 + 40 + 6 or 100 + 30 + 16

Solve problems in different contexts including missing number problems

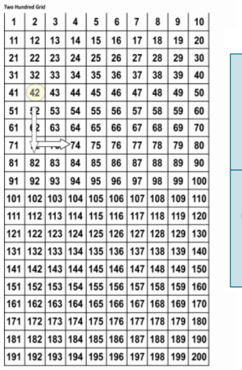
Year 4

Add numbers with up to four-digits (formal written column method) including numbers with up to two decimal places in the context of money

ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN** TWO NUMBERS INCLUDING DECIMALS, WITH DIFFERING NUMBERS OF DIGITS

Solve two-step problems in different contexts including missing number problems

Possible Concrete and Visual Representations



Two Hundred Grid

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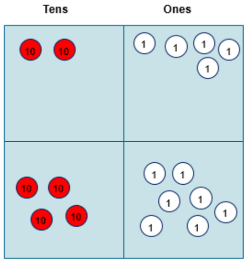
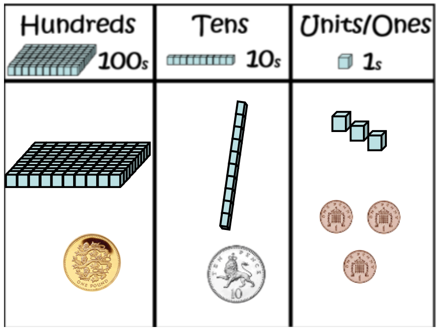
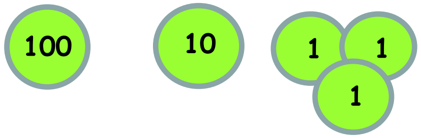
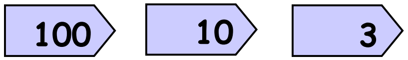
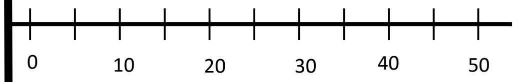


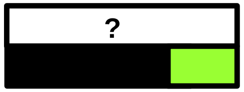


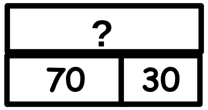






Partially numbered **and** blank number lines





Cuisenaire

Bar Model

Teacher Modelling/Children's Recording

Children apply, develop and secure their understanding of place value and begin to record in columns

Manipulatives SHOULD be used alongside algortihms

Column addition (no exchanging) with up to three-digits

$$\begin{array}{rcrcl} 40 & + & 1 & & \\ +\ 20 & + & 8 & & \\ \hline 60 & + & 9 & = & 69 \end{array}$$

Expanded recording without exchange

$$\begin{array}{rcrcl} 40 & + & 3 & & \\ 20 & + & 8 & & \\ \hline 70 & + & 1 & = & 71 \\ \hline 10 & & & & \end{array}$$

Expanded recording with exchange

$$\begin{array}{rcrcrcl} & 100 & + & 40 & + & 1 & \\ + & 100 & + & 20 & + & 8 & \\ \hline & 200 & + & 60 & + & 9 & = 269 \end{array}$$

Expanded recording

$$\begin{array}{r} 141 \\ +128 \\ \hline 9 \\ 60 \\ 200 \\ \hline 269 \end{array}$$

$$\begin{array}{r} \text{H T U} \\ 1\ 4\ 1 \\ +\ 1\ 2\ 8 \\ \hline 2\ 6\ 9 \\ \hline \end{array}$$

Compact (column) recording

$$\begin{array}{r} 1\ 4\ 3 \\ +\ 1\ 2\ 8 \\ \hline 2\ 7\ 1 \\ \hline 1\ \ \ \end{array}$$

Column addition (with exchanging)

$$\begin{array}{r} \text{H T U} \\ 7\ 8\ 9 \\ +\ 6\ 4\ 2 \\ \hline 1\ 4\ 3\ 1 \\ \hline 1\ \ 1\ \ \ \end{array}$$

$$\begin{array}{r} £\ 7.\ 8\ 9 \\ +\ £\ 6.\ 4\ 2 \\ \hline £\ 1\ 4.\ 3\ 1 \\ \hline 1\ \ \ 1\ \ \ \end{array}$$

Add decimals in the context of money

Compact (column) recording

Fluency

Count in ones, tens and hundreds maintaining fluency through varied and frequent practice

Count from 0 in multiples of 4, 8, 50 and 100

Find 10 or 100 more than a number

Mentally add HTU + ones, HTU + tens, HTU + hundreds

Perform mental calculations with two-digit numbers, the answer could exceed 100

Count in 6s, 7s, 9s, 25s and 100s

Find 1000 more than a number

Perform mental calculations with increasingly large numbers to aid fluency

Addition

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly. Addition and subtraction should be taught together.

End of Year Expectations

Year 5

Add numbers with more than four-digits and decimals up to three places

(formal written column method)

N.B. ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN** TWO NUMBERS INCLUDING DECIMALS, WITH DIFFERING NUMBERS OF DIGITS

Solve multi-step problems selecting and justifying methods

Perform mental calculations with increasingly large numbers

Year 6

Add numbers with more than four-digits and decimals up to three places

(formal written column method)

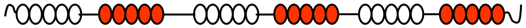
N.B. ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN** TWO NUMBERS, INCLUDING DECIMALS, WITH DIFFERING NUMBERS OF DIGITS

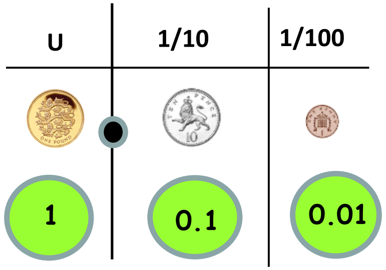
Solve more complex calculations mentally

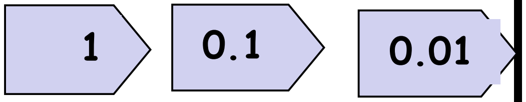
Solve multi-step problems in contexts, deciding which operations and methods to use and why

Possible Concrete and Visual Representations

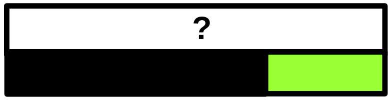
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	2	3	4	5	6	7	8	9



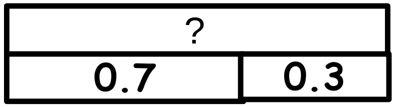


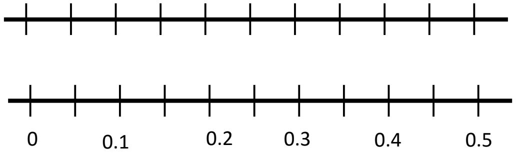


Cuisenaire

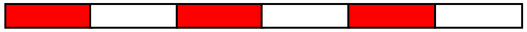


Bar Model





Partially numbered and blank number lines



Teacher Modelling/Children's Recording

Manipulatives could be used alongside algorithms

```
  2141        21.41
+ 1128     +   1.12
------         0.35
  3269     --------
           22.88
```

Column addition (no exchanging)

```
  5189        51.89
+ 3128     +   3.128
------     ---------
  8317        55.018
   11          1  1
```

Column addition (with exchanging)

Addition with decimals up to three decimal places including in different contexts e.g. money and measures

Fluency

Count forwards in powers of ten up to 100000

Count forwards in positive and negative whole numbers through zero

Practise mental calculations with increasingly large numbers

Practise fluency of written methods

Count in tens and hundreds increasing fluency of order and place value

Perform increasingly complex mental calculations and those with increasingly large numbers to aid fluency

Structures of Subtraction (Haylock and Cockburn 2008)

Children should experience problems with all the different subtraction structures in a range of practical and relevant contexts e.g. money and measurement

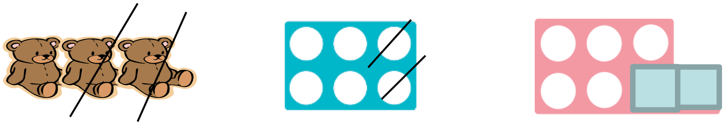
Partitioning

Take away

… how many left?

How many are not?

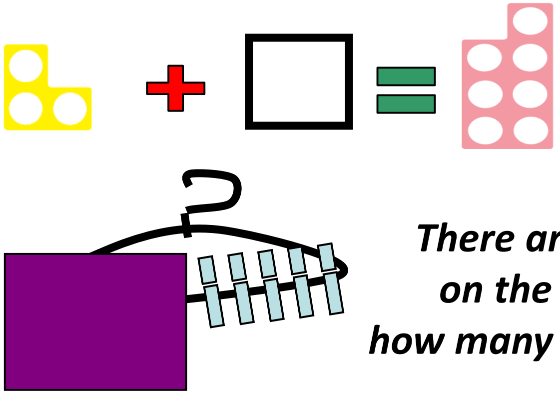
How many do not?



Inverse-of-addition

What must be added?

How many (much) more needed?



There are ten pegs on the hanger – how many are covered?

Comparison

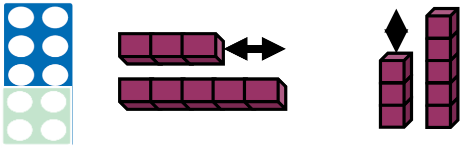
What is the difference?

How many more?

How many less (fewer)?

How much greater?

How much smaller?



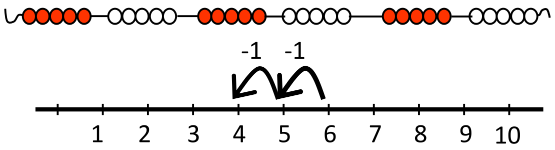
'two more than three is five or two less than five is three'

Reduction

Start at and reduce by

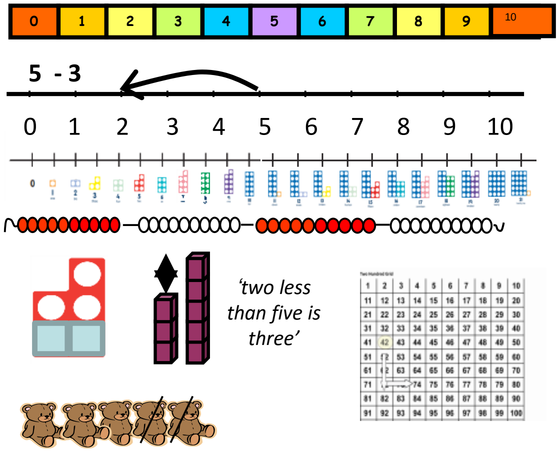
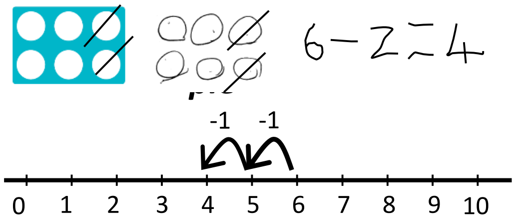
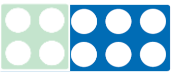
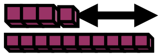
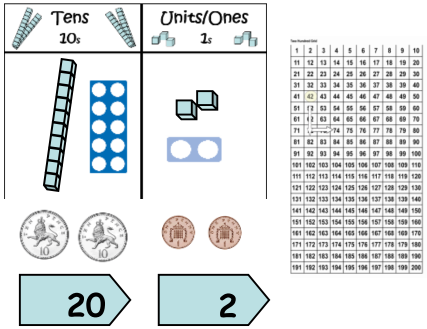
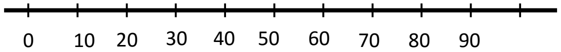
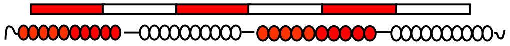
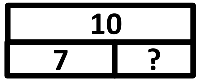
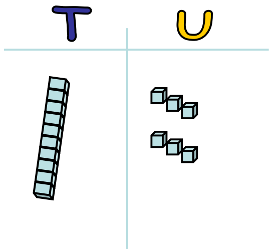
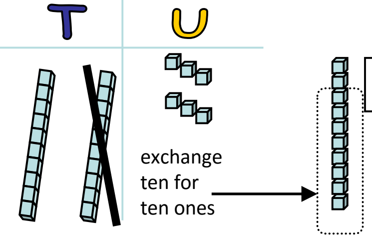
Count back by

Go down by



Subtraction

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly. Addition and subtraction should be taught together.

End of Year Expectations	Possible Concrete and Visual Representations	Children's Recording	Fluency
Year 1 Understand subtraction as taking away What is … less than …?) Compare quantities to say how many less and/or how many more	 'two less than five is three' Use practical resources such as bears, counters, cubes and number lines/hundred grids and progress to a resource such as Numicon to encourage counting back in groups rather than ones	Children may begin recording pictorially progressing to recording number sentences alongside  Children could use printed Numicon icons and stick these in, again progressing to recording number sentences alongside	Count backwards (including crossing 100) any given number Switch count between ones and tens e.g. 33, 32, 31, 30, 20, 10 Represent and use subtraction facts linked to number bonds up to 20 (establish addition and subtraction as related operations) Find one less than a number Find ten less than a number Count back in multiples of 2s, 5s and 10s starting on multiples to highlight pattern
Year 2 Understand subtraction as taking away **and** finding the difference Ensure children understand that subtraction is not commutative (can not be done in any order) Children should be able to partition numbers in different ways	 10 - 4 Finding the difference   Children should use concrete materials and pictorial representations, and use numbers in different contexts e,g, money and measures, Encourage children to partition numbers rather than counting in ones.  ***Numbered*** and ***partially numbered*** number lines   Cuisenaire Bar Model 	Children apply, develop and secure their understanding of place value and begin to record using jottings and number sentences  16 - 3 ***no exchanging***  exchange ten for ten ones 26 - 8 ***exchanging***	Practise addition and subtraction facts to 20 Show increasing fluency in deriving subtraction facts for numbers up to 10 and then up to 20 Use known facts to 20 to derive new facts e.g. 3 + 7 – 30 + 70 Use knowledge to derive and use subtraction number facts up to 100

Subtraction

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly. Addition and subtraction should be taught together.

End of Year Expectations	Possible Concrete and Visual Representations	Teacher Modelling/ Children's Recording	Fluency

Year 3

Subtract numbers with up to three-digits

(formal written column method)

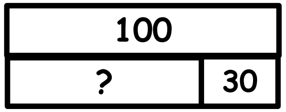
Children apply, develop and secure their understanding of place value and begin to record in columns

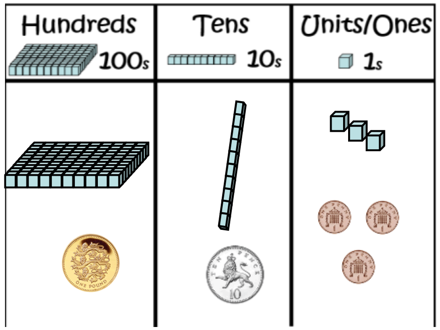

Two Hundred Grid

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

Cuisenaire

Bar Model





Children SHOULD use manipulatives alongside algorithms to transition between practical and abstract

no exchange

68 - 23

6 0 8

− 2 0 3

4 0 + 5 = 45

with exchange

63 - 28

$\cancel{50}$ 6 0 10 + 3

− 2 0 8

3 0 + 5 = 3 5

148 -121

1 0 0 4 0 8

− 1 0 0 2 0 1

0 + 2 0 + 7 = 2 7

Column subtraction (no exchange)

$$\begin{array}{r} 1\,4\,8 \\ -\ 1\,2\,1 \\ \hline 2\,7 \\ \hline \end{array}$$

Count back in ones, tens and hundreds maintaining fluency through varied and frequent practice

Switch count between hundreds, tens and ones e.g 500, 400, 300, 290, 280, 270, 269, 268, 267

Mentally subtract HTU + ones, HTU + tens, HTU + hundreds

Perform mental calculations with two-digit numbers

Find ten and a hundred less than a number with up to three-digits

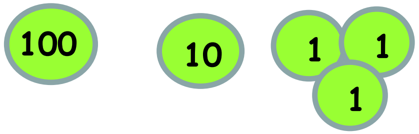
Year 4

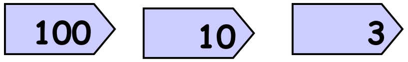
Subtract numbers with up to four-digits

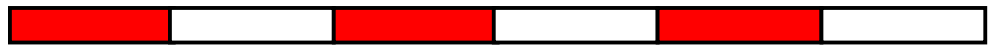
(formal written column method)

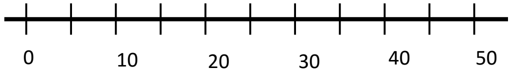
Understand subtraction as the inverse of addition

Solve two-step problems deciding upon the appropriate operations and methods and justifying choices made









Column subtraction (with exchange)

723 -317

$$\begin{array}{r} 7\ \ \overset{1}{\cancel{2}}\ \ {}^{1}3 \\ -\ 3\ \ 1\ \ 7 \\ \hline 4\ \ 0\ \ 6 \\ \hline \end{array}$$

723 -367

$$\begin{array}{r} \overset{6}{\cancel{7}}\ \ \overset{11}{\cancel{2}}\ \ {}^{1}3 \\ -\ 3\ \ 6\ \ 7 \\ \hline 3\ \ 5\ \ 6 \\ \hline \end{array}$$

$$\begin{array}{r} \overset{6}{\cancel{£7}}\,.\,\overset{11}{\cancel{2}}\ \ {}^{1}3 \\ -\ £3\,.\,6\ \ 7 \\ \hline £3\,.\,5\ \ 6 \\ \hline \end{array}$$

Ensure children can solve calculations where zero is a place holder

Count back in 6, 7, 9, 25 and 1000

Count back through zero to include negative numbers

Find 1000 less than a number

Continue to practise mental calculations with increasingly large numbers to aid fluency

Subtraction

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly. Addition and subtraction should be taught together.

End of Year Expectations

Year 5

Subtract larger numbers

(formal written column method)

N.B. ENSURE CHILDREN HAVE THE OPPORTUNITY TO SUBTRACT DECIMALS WITH DIFFERING NUMBERS OF DIGITS

Solve multi-step problems selecting and justifying methods

Subtract numbers mentally with increasingly large numbers

Year 6

Subtract multi-digit numbers including numbers with up to three decimal places

(formal written column method)

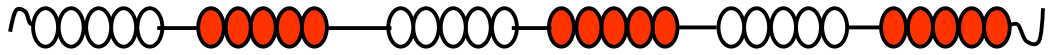
ENSURE CHILDREN HAVE THE OPPORTUNITY TO SUBTRACT DECIMALS, WITH DIFFERING NUMBERS OF DIGITS

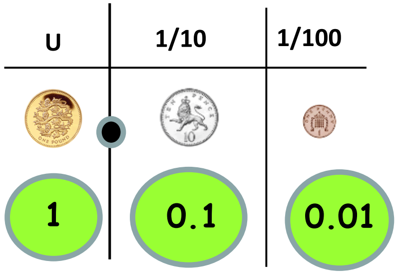
Solve multi-step problems in contexts, deciding which operations and methods to use and why

Solve more complex calculations mentally

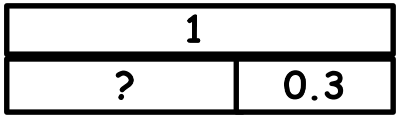
Possible Concrete and Visual Representations

0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	2	3	4	5	6	7	8	9

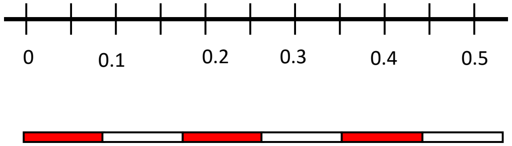




Cuisenaire



Bar Model



Teacher Modelling/ Children's Recording

Children might use manipulatives alongside algorithms

Column subtraction (no exchanging)

$$\begin{array}{r} 13548 \\ -\ 12128 \\ \hline 1420 \\ \hline \end{array}$$

Column subtraction (with exchanging)

$$\begin{array}{r} 1\,\overset{2}{\cancel{3}}\,\overset{13}{\cancel{4}}\,\overset{11}{\cancel{2}}\,\overset{1}{3} \\ -\ 1\,2\,6\,7\,8 \\ \hline 7\,4\,5 \\ \hline \end{array}$$

Ensure children can solve calculations where zero is a place holder

$$\begin{array}{r} 1.48 \\ -\ 1.21 \\ \hline 0.27 \\ \hline \end{array}$$

Column subtraction (no exchanging)

Column subtraction (with exchanging)

$$\begin{array}{r} \overset{6}{\cancel{7}}.\overset{11}{\cancel{2}}\overset{1}{3} \\ -\ 3.67 \\ \hline 3.56 \\ \hline \end{array}$$

Subtraction with decimals up to three decimal places including in different contexts e.g. money and measures

Fluency

Count backwards in powers of ten up to one million

Count backwards in positive and negative whole numbers through zero

Practise mental calculations with increasingly large numbers

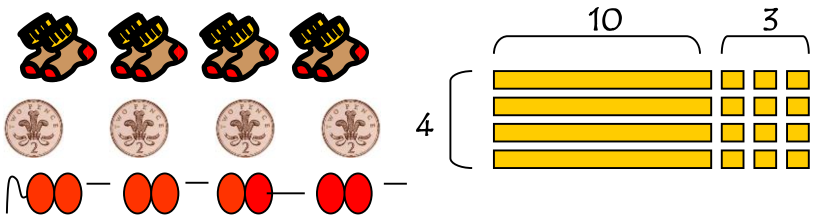
Undertake mental calculations with increasingly large numbers and more complex calculations

Structures of Multiplication (Haylock and Cockburn 2008)

Children should experience problems with all the different multiplication structures in a range of practical and relevant contexts e.g. money and measurement

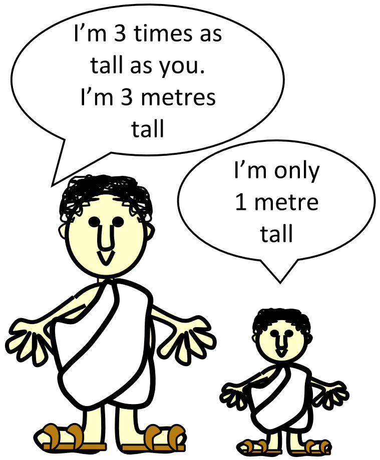
Repeated addition

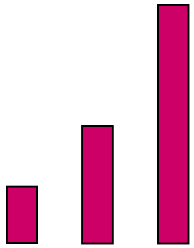
So many lots (sets) of so many
How many (how much) altogether
Per, each



Scaling

Scaling, scale factor
Doubling, trebling
So many times bigger than (longer than, heavier than, and so on)
So many times as much as (or as many as)





scaling with Cuisenaire

Commutative law

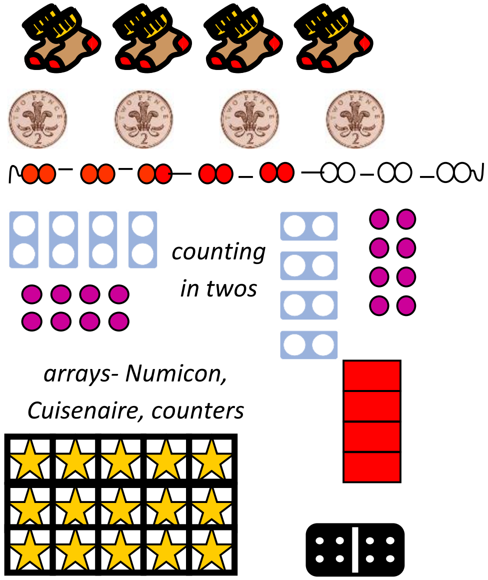
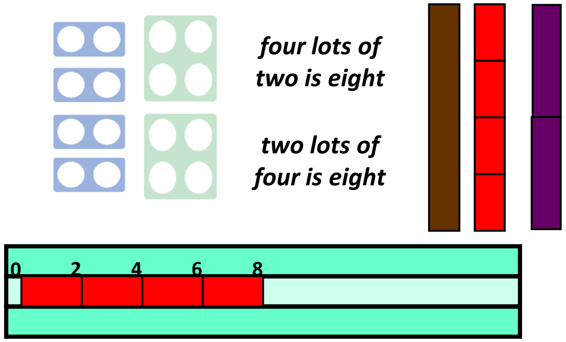
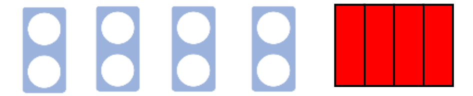
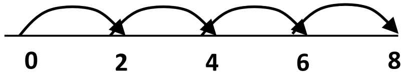
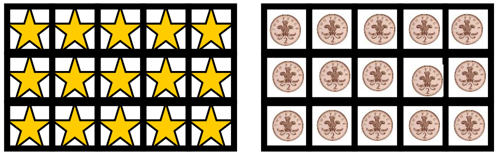
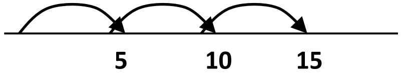
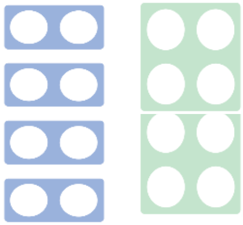
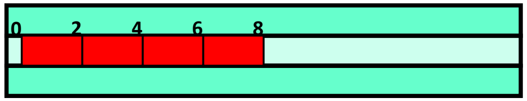
Scaling, scale factor
Doubling, trebling
So many times bigger than (longer than, heavier than, and so on)
So many times as much as (or as many as)

a x b and b x a are equal

4 x 2 is the same as/equal to 2 x 4

Multiplication

Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly.
Multiplication and division should be taught together.

End of Year Expectations	Possible concrete and visual representation	Children's Recording	Fluency
Year 1 Solve single step practical problems involving multiplication Use concrete objects, pictorial representations to explore grouping Make connections between arrays, number patterns and counting in twos, fives and tens Double numbers and quantities	 *counting in twos* *arrays- Numicon, Cuisenaire, counters* *flexible array*	Practical only e.g. link to small world Using concrete objects, pictorial representations and arrays with the support of an adult – take photographs/draw pictures – if using Numicon small icons could be stuck in  ***four lots of two is eight*** ***two lots of four is eight*** 0 2 4 6 8 *track with cuisenaire*	Count in twos, fives and tens from different multiples e.g. 6, 8, 10, 12 etc Emphasise number patterns Double number and quantities
Year 2 Understand multiplication as repeated addition Calculate mathematical statements for multiplication within the tables and write them using symbols Understand and solve problems involving arrays Ensure children understand that multiplication is commutative (can be done in any order) Understand that multiplication and division are inverse operations	 2 + 2 + 2 + 2 = 4 x 2 ***two add two add two add two add two = four lots of two***  0 2 4 6 8  *flexible array*  5 10 15	Record practical work as number sentences  ***4 x 2 = 8*** ***2 x 4 = 8***   0 2 4 6 8	Count in twos, threes, fives from zero and tens from any number e.g. 6, 8, 10, 12 etc Emphasise number patterns Introduction to multiplication tables. Practise to become fluent in multiplication facts for 2, 5 and 10 Solve multiplication problems mentally

Multiplication – multiplication and division should be taught together– refer to structures of multiplication

End of Year Expectations

Year 3

Develop reliable written methods

Understand and solve scaling problems

Solve problems involving multiplication including correspondence

Year 4

Multiplying three numbers

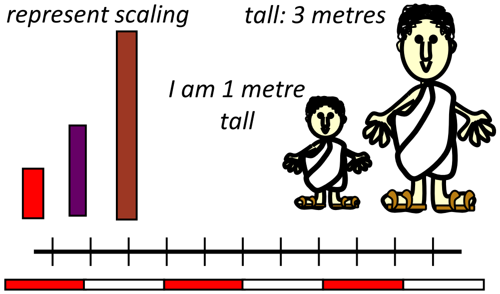
Solve two-step problems

Multiplying by 0 and by 1

Develop fluency in short multiplication using formal written layout

Solve problems involving multiplication including using the distributive law, integer scaling problems and harder correspondence problems

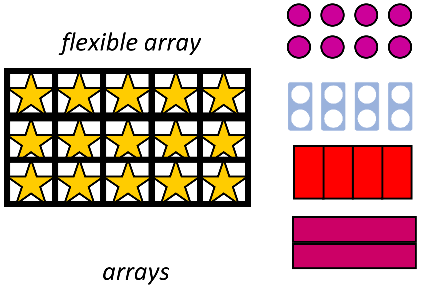
Possible concrete and visual representation



Cuisenaire to represent scaling

Statue is 3 times as tall: 3 metres

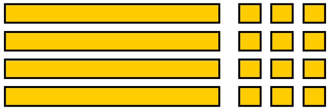
I am 1 metre tall

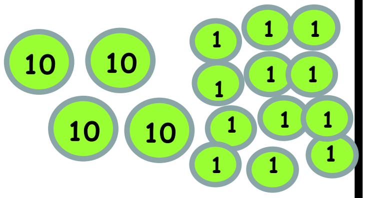


flexible array

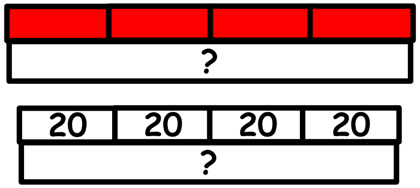
arrays

4 × 13





place value counters

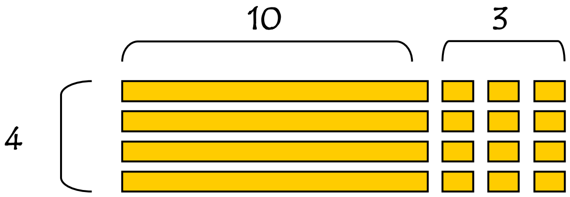


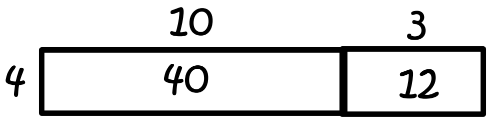
bar models

Teacher Modelling/Children's Recording

Children must use manipulatives alongside algorithms

4 x 13 'four lots of thirteen'





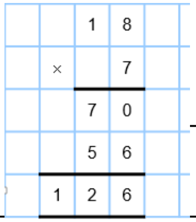
Expanded methods – grid and area

x	10	3
4	40	12

40 + 12 = 52

Progressing to developing fluency in short multiplication

```
  1 3
x   4
-----
  5 2
-----
  1
```



		1	8		
	×		7		
		7	0		
		5	6		
	1	2	6		

```
1 3 3
x   4
-----
5 3 2
-----
1 1
```

Start with digits that are below five so children can practise method without encountering difficulty with multiplication tables

Fluency

Count from 0 in multiples of 4, 8, 50 and 100

Use multiples of 2, 3, 4, 5, 8, 10, 50 and 100

Practise mental recall of multiplication tables – 3, 4 and 8x times tables

Connect the 2, 4 and 8 times tables using doubling

Develop efficient mental methods using commutativity and multiplication facts to derive related facts e.g. 4 x 4 x 12 = 12 x 4 x 5 = 12 x 20

Count in multiples of 6, 7, 9, 25 and 1000

Recall and use multiplication facts up to 12 x 12 with increasing fluency

Derive multiplication facts with up to three-digits

Recognise and use factor pairs and commutativity in mental calculations

Use the distributive law

Combine knowledge of number facts and rules of arithmetic to solve mental and written calculations e.g. 2 x 6 x 5 = 10 x 6

Multiplication - multiplication and division should be taught together– refer to structures of multiplication

End of Year Expectations

Year 5

Multiply decimals with up to three decimal places

Identify multiples and factors including finding all factor pairs of a number, and common factors of two numbers

Solve problems involving all four operations where larger numbers are used by decomposing them into their factors

Multiply whole numbers and those involving decimals by 10, 100 & 1000

Understand and use multiplication and division as inverses including in problems involving missing numbers and balancing equations

Solve problems involving multiplication and division including scaling by simple fractions

Know and use the vocabulary of prime numbers, prime factors and composite (non-prime)

Recognise and square and cube numbers and associated notation

Year 6

Multiply numbers up to 4-digit x TU

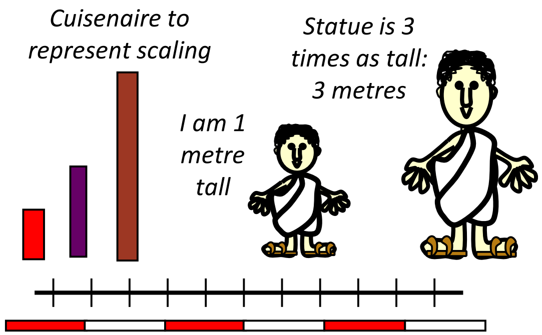
Multiply numbers with up to two decimal places x whole number

Multiply multi-digit numbers up to four-digits by a two-digit whole number

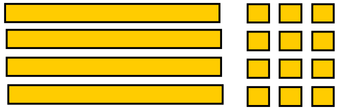
Multiply single –digit numbers with up to two-decimal places by whole numbers

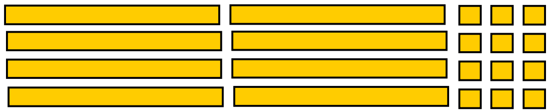
Solve problems involving all four operations

Possible concrete and visual representation

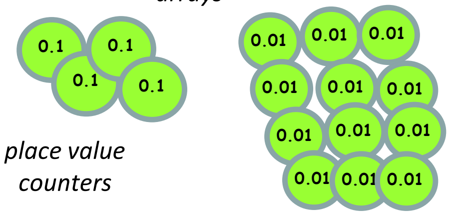



flexible array

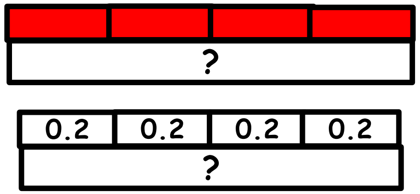

4 x 13


4 x 23

arrays



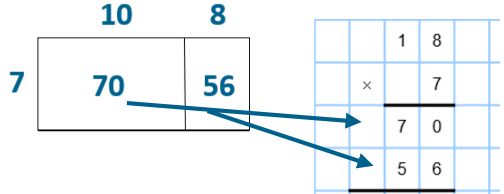
place value counters



bar models

Teacher Modelling/Children’s Recording

Children might use manipulatives alongside algorithms

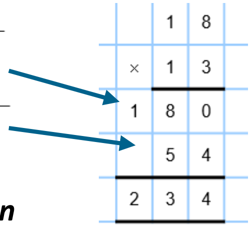


Short multiplication

1 3 2 4
x 6
7 9 4 4
1 1 2

3 . 2 4
x 6
1 9 . 4 4
1 2

x	10	8
10	100	80
3	30	24



Long multiplication

1 3 2 4
x 2 6
7 9 4 4
2 6 4 8 0
1 1 2
3 4 4 2 4
1 1 1

3 . 2 4
x 2 6
1 9 . 4 4
6 4 . 8 0
1 2
8 4 . 2 4
1 1

Fluency

Count forwards in steps of powers of 10 from any given number up to 1 000 000

Practise and extend use of formal written method of short multiplication

Apply all multiplication tables frequently. Commit them to memory and use them confidently to make larger calculations

Multiply numbers mentally drawing upon known facts

Undertake mental calculations with increasingly large numbers

Continue to use all multiplication tables to calculate mathematical statements in order to maintain fluency

Structures for Division (Haylock and Cockburn 2008)

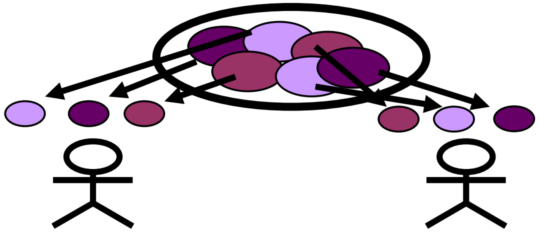
Children should experience problems with the different division structures in a range of practical and relevant contexts e.g. money and measurement

Equal-sharing

Sharing equally between
How many (much) each?

6 shared equally by 2

6 ÷ 2

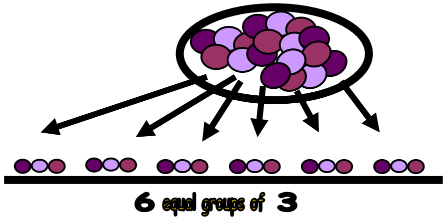


Inverse of multiplication (Grouping)

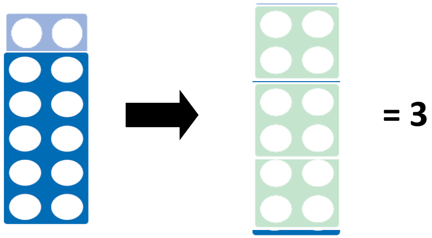
So many lots (sets/groups) of so many
Share equally in to groups of …

18 ÷ 3

18 divided into equal groups of 3s



Divide twelve into equal groups of four



Make 12

Overlay groups of four

Ratio structure

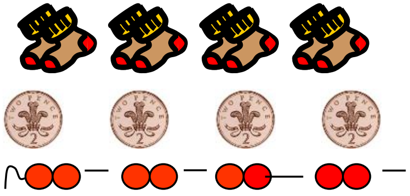
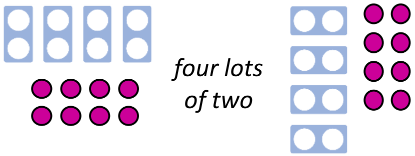
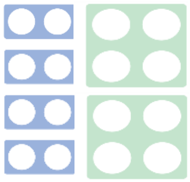
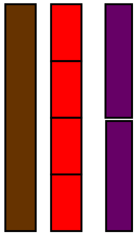
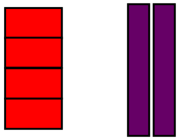
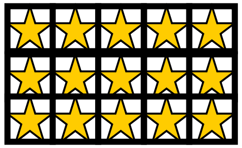
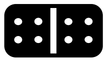
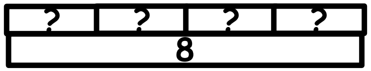
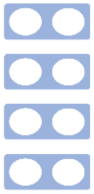
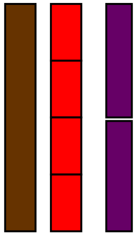
comparison
inverse of scaling structure of multiplication
scale factor (decrease)

Barney earns three times more than Fred. If Barney earns £900 how much does Fred earn?

Jo's journey to school is three times as long as Ella's. If Jo walks to school in 30 minutes how long does it take Ella?

Division

Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly. Multiplication and division should be taught together.

End of Year Expectations	Possible concrete and visual representation	Teacher Modelling/Children's Recording	Fluency
Year 1 Solve single step practical problems involving division Use concrete objects, pictorial representations Understand division as grouping and sharing Use the language of 'sharing equally between'	*counting in groups of twos*  *straw bundles*   *Numicon and counter arrays*	Practical only e.g. link to small world Using concrete objects, pictorial representations and arrays with the support of an adult – take photographs/draw pictures – if using Numicon small icons could be stuck in  ***Eight can be divided into four equal groups of two or two equal groups of four*** 	Count in twos, fives and tens from different multiples e.g. 6, 8, 10, 12 etc Emphasise patterns Find simple fractions eg half and quarter, of objects, numbers and quantities
Year 2 Solve single step practical problems involving division Use concrete objects, pictorial representations Understand division as grouping Find halves and then quarters Work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete quantities and to arrays	 *Cuisenaire* *four lots of two* *two lots of four*  *flexible array*  *doubling*  bar models  	Record as number sentences using ÷ and = 8 ÷ 4 Eight divided into four equal groups = two in each group 8 ÷ 4 = 2   ***Eight can be divided into four equal groups of two or two equal groups of four***	Count back in twos, threes, fives from zero and tens from any number e.g. 12, 10, 8, 6 etc Emphasise patterns Connect ten times table to place value and five times table to divisions on a clock face Introduction to multiplication tables. Practise to become fluent in division facts for 2, 5 and 10 Solve division problems involving grouping and sharing

Division - multiplication and division should be taught together– refer to structures of division

End of Year Expectations

Year 3

Develop a reliable written method for division

Solve problems involving missing numbers

Solve problems including those that involve scaling

Recognise, find and name ½ and ¼ of an object, shape or quantity

Understand the link between unit fractions and division

Connect 1/10 to division by 10

Count in tenths

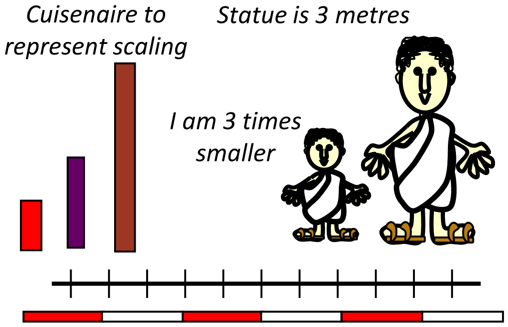
Year 4

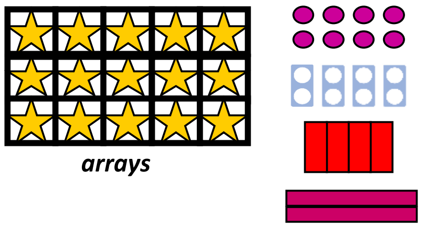
Become fluent in the formal written method of short division with exact answers when dividing by a one-digit number

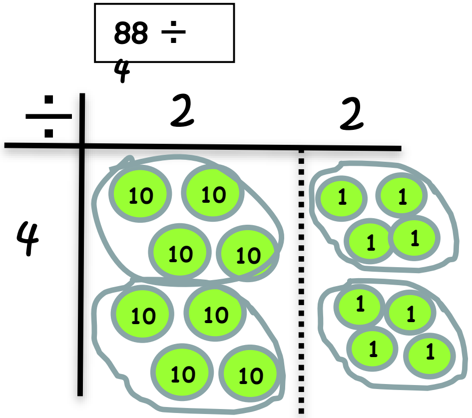
Divide one- or two-digit numbers by 10 or 100, identifying value of digits as tenths or hundredths

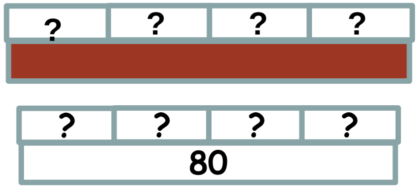
Solve two-step problems in different contexts, choosing the appropriate operation, working with increasingly harder numbers including correspondence questions e.g. three cakes shared equally between 10 children

Possible concrete and visual representation




arrays

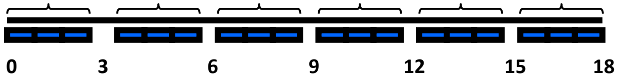




bar models

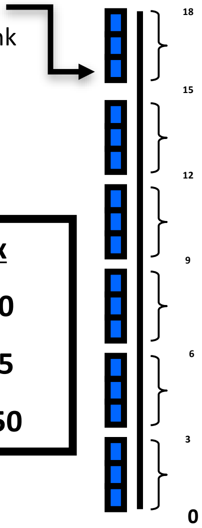
Teacher Modelling/Children's Recording

Children <u>should</u> use manipulatives alongside algorithms



Repeated subtraction - chunking

Ensure children see/understand the link between grouping on a number line and vertical recording for chunking



95 ÷ 5 = 19

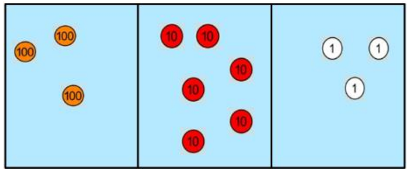
```
  95
- 50   ( 10 x 5 )
  45
- 25   ( 5 x 5 )
  20
- 20   ( 4 x 5 )
   0
```

Fact Box

2 x 5 = 10

5 x 5 = 25

10 x 5 = 50



```
 ÷ | 1  2  1
---+---------
 3 | 3  6  3
```

321 ÷ 3

Short division- no remainders

560 ÷ 4

```
 ÷ | 1  4  0
---+----------
 4 | 5 ¹6  0
```

Fluency

Recall and use related division facts for the 3, 4 and 8x tables (Continue to practise other tables)

Write and calculate mathematical statements for division using what is known

Use division facts to derive related division facts e.g. using 6 ÷ 3 = 2 to work out 60 ÷ 3 = 20

Continue to practise recalling division facts for multiplication tables up to 12 x 12

Practise mental methods and extend this to three-digit numbers for example 200 x 3 = 600 into 600 ÷ 3 = 200

Use place value, known and derived facts to divide mentally, including dividing by 1

Recognise and use factor pairs and commutativity in mental calculations

Division - multiplication and division should be taught together– refer to structures of division

End of Year Expectations

Year 5

Identify factors , including finding all factor pairs of a number, and common factors of two numbers

Practise and extend the formal written method of short division: numbers up to four-digits by a one-digit number

Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding as appropriate for the context

Use multiplication and division as inverses

Solve problems involving division including scaling down

Divide whole numbers and those involving decimals by 10, 100 & 1000

Year 6

Divide numbers up to 4-digits by a 2-digit whole number using formal written methods of long division, interpret remainders as whole numbers, fractions or by rounding, as appropriate for the context

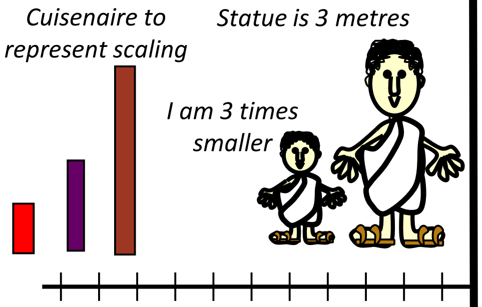
Divide numbers with up to 2 decimal places by 1-digit and 2-digit whole numbers, initially in practical contexts involving money and measures

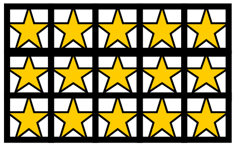
Understand the relationship between unit fractions and division

Recognise division calculations as the inverse of multiplication

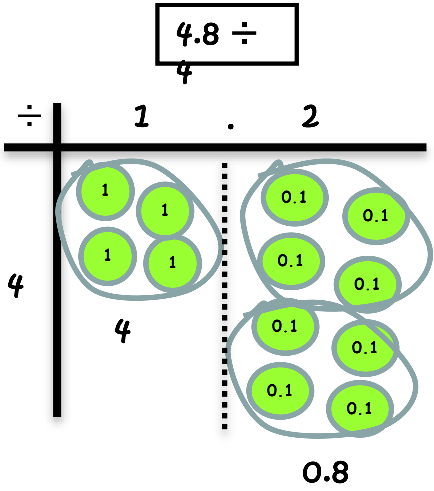
Solve problems involving division

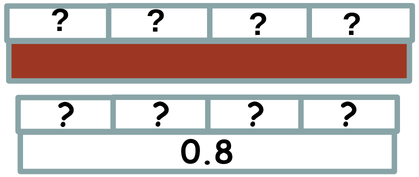
Possible concrete and visual representation





flexible arrays





bar models

Teacher Modelling/Children's Recording

Children might use manipulatives alongside algorithms

short division

without remainder

560 ÷ 4

$$\begin{array}{r|l} \multicolumn{1}{r}{} & 1\;4\;0 \\ \cline{2-2} 4 & 5\;{}^{1}6\;0 \end{array}$$

remainder as a decimal

564 ÷ 5

$$\begin{array}{r|l} \multicolumn{1}{r}{} & 1\;1\;2\;.\;8 \\ \cline{2-2} 5 & 5\;6\;{}^{1}4\;.{}^{4}0 \end{array}$$

remainder as a fraction

$$\begin{array}{r|l} \multicolumn{1}{r}{} & 1\;1\;2\;\;2/5 \\ \cline{2-2} 5 & 5\;6\;{}^{1}4 \end{array}$$

long division

560 ÷ 24

$$\begin{array}{r|l} \multicolumn{1}{r}{} & 2\;3\;\text{r}\,8 \\ \cline{2-2} 24 & 5\;6\;0 \\ \multicolumn{1}{r}{} & -4\;8\downarrow \\ \cline{2-2} \multicolumn{1}{r}{} & \;\;\;8\;0 \\ \multicolumn{1}{r}{} & \;-7\;2 \\ \cline{2-2} \multicolumn{1}{r}{} & \;\;\;\;\;\;8 \end{array}$$

remainder as a whole number

$$\begin{array}{r|l} \multicolumn{1}{r}{} & 2\;3\;\;8/24\;(1/3) \\ \cline{2-2} 24 & 5\;6\;0 \\ \multicolumn{1}{r}{} & -4\;8\downarrow \\ \cline{2-2} \multicolumn{1}{r}{} & \;\;\;8\;0 \\ \multicolumn{1}{r}{} & \;\;\;7\;2 \\ \cline{2-2} \multicolumn{1}{r}{} & \;\;\;\;\;\;8 \end{array}$$

remainder as a fraction in its lowest form

$$\begin{array}{r|l} \multicolumn{1}{r}{} & 2\;3\;.\;3 \\ \cline{2-2} 24 & 5\;6\;0\;.\;0 \\ \multicolumn{1}{r}{} & -4\;8\downarrow \\ \cline{2-2} \multicolumn{1}{r}{} & \;\;\;8\;0 \\ \multicolumn{1}{r}{} & \;\;\;7\;2 \\ \cline{2-2} \multicolumn{1}{r}{} & \;\;\;\;\;\;8\;0 \\ \multicolumn{1}{r}{} & \;\;\;\;\;\;7\;2 \end{array}$$

remainder as a decimal

Fluency

Count backwards in steps of powers of 10 for any given number up to 1 000 000

Count backwards with positive/negative whole numbers through zero

Practise mental calculation with increasingly large numbers

Apply all multiplication tables and related division facts frequently, commit them to memory and use them to confidently to make larger calculations

Practise division for larger numbers, using the formal written methods of short and long division

Continue to use all multiplication tables and division facts to maintain fluency

Perform mental calculations, including with mixed operations and larger numbers